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PROPAGATING BUCKLES IN LONG CONFINED CYLINDRICAL SHELLS

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Abstract—In many practical engineering applications, relatively stiff, long, circular cavities are lined with more compliant liner shells. Pressure applied through the porous walls of the confining medium can cause buckling of the shells. Such buckling is usually local in nature and occurs at a section with the biggest geometric imperfection. The paper presents experimental evidence which demonstrates that, once such a shell has been locally dented, a buckle which propagates within the confines of the cavity can be initiated. Such a buckle has the potential of completely collapsing the liner. The lowest pressure at which this buckle will propagate is established experimentally through a parametric study of the problem. The phenomenon is found to be physically similar to the propagating buckle problem which can develop in offshore pipelines. A difference is that in the case of the confined shell, the instability is shown to have a strong geometric dependence and, as a result, it can be developed in the case of thin elastic as well as elasto-plastic shells.

INTRODUCTION

In a number of practical engineering applications, long, thin-walled metal shells are used as liners of relatively stiff circular cylindrical cavities. In most of these applications, the liner is designed to be in contact with the cavity wall. Examples of such applications are: oil, gas, steam and water well casing; steel lining for tunnels and ducts used for transporting gases and liquids at hydroelectric plants, power stations, nuclear reactor plants, etc.; steel pipes cladded with a thin layer of noncorrosive metal to protect the main structure from the corrosive fluids it transports, etc. In all these applications, conditions can develop that lead to buckling of the liner which is exhibited in the form of a local inward deformation. Although the conditions for buckling vary from application to application, differential expansion of the liner and the cavity material and the build up of pressure between the confining cavity and the shell seem to represent the two main causes of local buckling. For the case of a cladded pipe or other applications of similar geometry, buckling of the thin liner can also result from severe deformations of the outside shell.

A number of investigators[1–4] idealized local buckling of such confined shells as a thin-walled, elastic ring in a rigid cavity and proceeded to establish engineering type design criteria to ensure the safety of liners from pressure-induced buckling. A great deal of insight into this class of problems can also be gained from studies of the stability of confined rings under thermal and inertial loads[5–10]. It was shown that the load-deflection response of such rings exhibits a limit load type of instability which is a function of the geometry of initial imperfections that may exist. More recent studies of confined shells[11, 12] and rings[13] under external pressure, have shown the stability of the pressurized confined structure to be similarly affected by the presence of initial geometric imperfections.

The main conclusion drawn from these studies is that given geometric imperfections, which are always present in the types of structures discussed here, buckling can occur which will result in a localized separation of the liner from the wall. Similar liner damages can also be caused by ground formation movements. These can be due to time-dependent settlement of the ground or due to earthquakes or other transient causes of ground movements.

This paper addresses the question of what the consequences would be, should hydrostatic pressure somehow develop between the confining wall and a locally damaged confined shell. Experiments carried out on model shells have revealed that conditions exist under

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which the liner can develop a buckle that propagates. The buckle gets initiated from the weakened section of the shell in the region of a local buckle or damage, and propagates upstream and downstream of that point. Given a source of pressure, the complete length of shell can potentially be destroyed.

The existence of the phenomenon will be demonstrated experimentally. It will also be shown that a well-defined pressure exists which has to be exceeded before the buckle propagates. This critical characteristic pressure of the shell has been given the name "Confined Propagation Pressure" (P_{pc}). An experimental parametric study of this pressure is presented.

The "Confined Propagating Buckle" has great similarities with the problem of the propagating buckle in offshore pipelines studied by a number of investigators[14–19] in the past few years. The similarities and differences of the two problems are pointed out and discussed.

THE PROBLEM

The purpose of a liner shell varies from application to application. As a result, the diameter-to-thickness ratio of interest also varies from 500, for certain large tunnels in power stations, down to 10 for casing of very deep oil wells. A short description of some of the applications is given below in the way of motivating the problem and also guiding the reader through the assumptions and idealizations made in both the experimental and analytical work that follows.

In the case of well casing, the steel liner's main purpose is to maintain the integrity of the circular geometry of the well. It is designed to protect the operational tubes of the well inside it from creep of the surrounding ground, falling debris, retreating ground formations, and external pressure from rising water levels or gas from the ground around the well. The actual load conditions faced by the casing are very difficult to predict and vary from well to well. Casing is thus typically designed to resist collapse under hydrostatic pressure equivalent to a water head equal to the depth.

Wells vary in diameter from 10 to 36 in. (0.25–0.90 m) for water wells and from 8 to 18 in. (0.20–0.45 m) for oil and gas wells. Typically, the casing is 1–2 in. (25–30 mm) smaller in diameter than the nominal size of the hole. Creeping formations, falling debris, etc., can close the original gap in many places along the length of the well. In many wells, the casing is grouted with cement either along the whole or part of its length. This usually consists of forcing cement into the gap between the casing and the wall. As expected, grouted casing has higher resistance to buckling by external pressure[20] than ungrouted casing.

Texter[21] and other investigators have reported collapse failures of casings. According to Texter, two typical failure modes have been observed in the field. He calls them the "ribbon" and "trough" modes. The "ribbon" is a "dogbone" ($n = 2$) mode of collapse commonly obtained in collapse and collapse propagation experiments on tubes under external pressure. An example of the "trough" mode, as it affected a string of casing 60 ft (18 m) long, is shown in Fig. 1 (reproduced from Texter's paper[21], courtesy API). It has the typical U-shaped collapsed cross-section common to cylindrical shells buckled in a cylindrical confining cavity. Similar collapse failures, in which "several" strings of casing were found collapsed, have been reported by others. The reported "large length" of collapsed casing involved suggests that perhaps something more than just classical buckling of the structure may have been involved in these failures. An alternative possible scenario will be suggested in the light of the experimental observations that follow.

The second class of problems lies at the other extreme of the D/t range. It concerns lining shells for long, large-diameter underground shafts constructed at large power stations. Reference [2] gives a good review of these applications. Due to the large diameter (several meters across) of these structures, it has been found more economical to drill circular tunnels into the ground. The tunnels are lined with a thin-walled ($D/t \sim 300$–500) steel shell. The gap between the tunnel wall and the liner is filled with concrete grout. Thus, the liner's main purpose is to contain the flow. The loads caused by the internal pressure are reacted mainly by the concrete and the rock around it. Although the shafts usually operate

Fig. 1. Collapsed string of well casing (Texter (1955), courtesy, Am. Petr. Inst.).

under internal pressure, they are often dewatered. During this process, and due to the porous nature of the rocks and the presence of cracks in the concrete, pressure can develop between the liner and the confining cavity, which can lead to buckling. Similar but smaller in diameter lined ducts were reported to have experienced buckling failures in [1, 11, 12].

As in the case of the casing, it will be shown again that, if continuous flow of water at a minimum pressure is available, such local buckles can initiate much more catastrophic collapse failures.

Cladded pipes are very similar structures. External pressure can be exerted on the thin-walled liner by corrosive gases which form between the shell and the liner.

With this short review of some of the applications, we proceed to demonstrate that such shells can develop a buckle which propagates and identify some of the characteristics of the problem.

THE CONFINED PROPAGATING BUCKLE

In order to demonstrate the phenomenon, it was necessary first to establish experimental conditions which simulate those that exist in the problems outlined above. The problem geometry can be idealized as a long, circular cylindrical shell surrounded by a relatively stiff contacting medium. Drawn, seamless, metal tubes with diameters ranging from 1.25 in. (32 mm) to 4.0 in. (102 mm) were used to model the liners. A solid cylinder of plaster of paris was molded around each tube in the following fashion. A thick steel tube cut in two halves along a generator was used as the mold. The final assembly, consisting of the liner tube, the plaster and the steel mold, is shown schematically in Fig. 2. The tube was well lubricated prior to pouring the plaster in order to reduce the friction between the plaster and the tube wall. Postmortem examination of the tube and plaster indicated that this molding procedure yielded contact with a minimum of friction.

Plaster of paris was selected because it retains its strength in water and dries much faster than cement (about 12 h). The hardness of this type of plaster is quite high. Results from a comparative experiment using cement showed no quantifiable effect on the measured variables from the two grout materials. The steel mold was left in place to ensure that no cracking of the plaster occurred during the experiment.

Since the goal of this study was to demonstrate the phenomenon and establish the confined propagation pressure, no attempt was made to simulate the initiation process as it occurs in practice. Instead, it was opted to initiate the buckle on a section of tube left outside the confinement by physically denting it, as shown in Fig. 2. The confined length of the test specimen was always longer than 25 tube diameters. The unconfined section was 5–15 diameters long.

Experimental procedure

The experiments were conducted under volume control conditions in order to ensure quasistatic propagation conditions. The assembly of tube, plaster and steel mold shown in Fig. 2, was placed in a high-pressure testing facility. The facility had an internal diameter of 7 in. (0.178 m) and a working pressure of 10,000 psi (690 bar). The vessel was completely filled with water and pressurized using a positive displacement water pump. The specimen was closed at both ends, but the inside of the tube was vented directly to the atmosphere to ensure that the internal pressure was kept at the same reference level during the collapse process.

During the experiment, the pressure in the tank was monitored with a pressure transducer. A typical time–pressure history of the experiments is shown schematically in Fig. 3. The section of pipe outside the confinement collapsed first. The collapse process was initiated from the inflicted damage. The value of pressure P_I represents the initiation pressure for that imperfection. During the initiation process, completed by T_2, the damaged section locally deforms into the profile a propagating buckle propagates (see [16]). This profile is one that gradually transforms the originally circular tube cross-section into a "dogbone" shaped collapsed section.

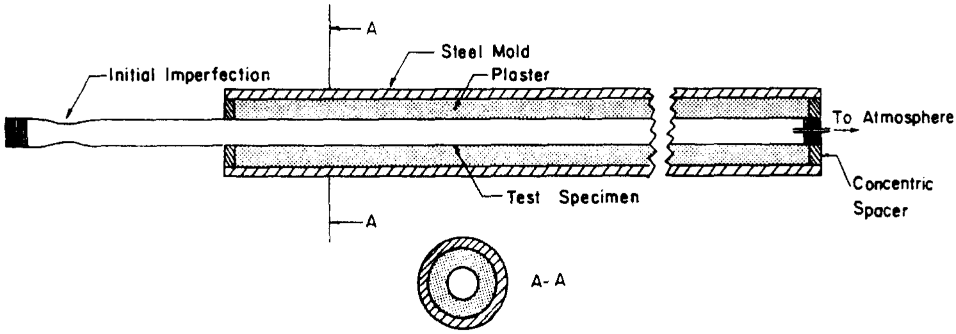


Fig. 2. Test specimen and confinement assembly.

Beyond T_2, the buckle propagates at a constant pressure which represents the propagation pressure (P_p) of the given tube. Propagation ceases once the buckle has reached the edge of the confinement. If pumping water into the vessel is continued, a sharp rise in pressure is experienced. The rise is not instantaneous, because a relatively small volume of water is taken to flatten the already collapsed section of pipe further. The confined section of pipe remains practically undisturbed until time T_4. (In fact, water may creep between the interface of the tube and plaster and pressurize the tube to a certain degree, but this does not have any effect on the events described here.)

At some pressure, P_{IC} (initiation pressure of confined propagation) the flattened section of tube adjacent to the beginning of the confinement snaps into a U shape enabling the buckle to start penetrating the confinement. Once this profile is fully developed, steady state conditions are quickly established (typically in less than 5 diameters from the edge of the confinement). The pressure stabilizes at a new plateau at which the whole length of the confined tube can be collapsed. The value of this pressure has been given the name "Confined Propagation Pressure" (P_{pc}).

The rate at which the confined buckle propagates is, at all times, controlled by the rate

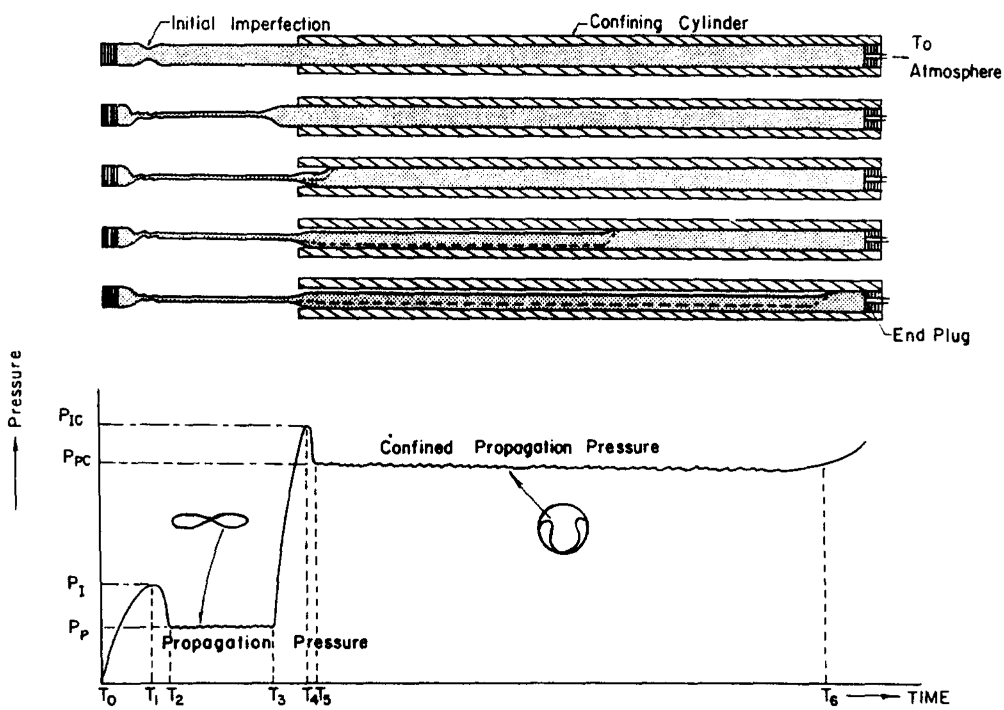


Fig. 3. Schematic representation of a typical quasistatic confined propagation experiment.

at which water is pumped into the pressure vessel. This rate was kept at a low enough level to ensure that only 2–3 diameters of the tube length were collapsed per min.

Figures 4(a) and (b) show a postmortem examination of the test specimen. The geometric integrity of the plaster is clearly seen to be retained.

Figure 5 shows a picture of the profile of propagation of the confined propagating buckle. The profile transforms the originally circular cross section of the tube into a U-shaped collapsed section (the confinement was taken away). The length of this profile is typically 3–5 diameters. Figure 6 shows slices of a tube cut through the profile indicating the sequence of configurations involved in the collapse process.

A separate experiment was carried out in order to obtain visible evidence of the collapse process in progress. A 1.5 in. (38 mm) hole was bored into a long transparent acrylic rod. The rod had a diameter of 4 in. (0.10 m). A thin-walled tube was fitted into the cavity. [A gap of maximum size of 0.002 in. (0.05 mm) had to be allowed between the acrylic and the tube.] The confined tube was placed in a transparent pressure vessel, and a buckle was initiated and propagated using the procedure described above. The progress of the buckle was recorded photographically during propagation. Edited results from this experiment are assembled in Fig. 7. The development of the steady state propagation process is clearly visible. Due to the imperfect contact that existed in this experiment, the recorded confined propagation process will not be quoted here. It should, however, be pointed out that gaps between the cavity wall and the tube tend to lower P_{pc}. The magnitude of this effect has not been quantified at this stage.

In completing this section, it is again pointed out that the particular confined buckle initiation process, chosen in the experimental procedure described, is not one that represents well the way buckles are initiated in the practical applications described in the previous section. As already mentioned, most such buckles would be initiated from within the confines of the cavity. It is, however, emphasized that the steady state confined propagation process, the identification of which was the main purpose of this study, is independent of the initiation process. The results that follow should be viewed in the spirit of this comment.

Experimental results

A series of experiments were carried out using commercially available seamless, drawn tubes. Two materials were used, aluminum 6061-T6 and stainless steel 304. For each material, experimental results were obtained for a number of different diameter-to-thickness ratios (D/t) between approximately 20 and 100. Longitudinal test specimens cut from the tubes were used to obtain stress–strain curves for each tube tested. Wall thickness variations of max $|\Delta t/t| \cong 8\%$ existed in the tubes tested. As a result, the confined propagation pressure was occasionally position dependent. This source of error was minimized by using mean values for D and t in the reduction of the data. The measured confined propagation pressures of the two groups of test specimens are listed in Tables 1 and 2 together with their material and geometric parameters. The stress–strain curves of the test specimens of each material group had approximately the same elastic modulus (E) and hardening parameter (n) but different yield stress (σ_0) (see Table 3). In view of the heavy plastic deformation involved in the collapse process, this difference can be important. Thus, the propagation pressure was normalized by the measured yield stress (σ_0—yield stress as defined by the 0.2% strain offset) of each tube.

The experimental results are plotted as a function of the tube D/t on logarithmic scales in Fig. 8. An attempt is made to fit these with a power law represented by the least squares straight lines shown, i.e.

$$\frac{P_{pc}}{\sigma_0} = A\left(\frac{t}{D}\right)^{\beta}. \tag{1}$$

The aluminum results can be seen to be well represented by this relationship except for D/t values beyond 100. In the case of stainless steel, the scatter in the results is much bigger. In addition, for D/t values beyond approximately 60, the results seem to deviate from the

Table 1. Aluminum test specimen properties and confined propagation pressures measured

	D in. (mm)	$\frac{D}{t}$	P_{pc} psi (bar†)	$\frac{P_{pc}}{\sigma_0} \times 10^3$
1	1.753 (44.53)	21.12	1830 (126.21)	37.35
2	1.498 (38.05)	25.82	1080 (74.48)	24.0
3	1.748 (44.40)	26.89	1005 (69.31)	22.33
4	1.124 (28.55)	32.11	659 (45.45)	14.64
5	1.250 (31.75)	35.71	555 (38.28)	12.33
6	1.003 (25.48)	50.15	261 (18.00)	5.80
7	1.497 (38.02)	53.46	219 (15.10)	4.87
8	1.496 (38.00)	69.58	132 (9.10)	2.93
9	2.75 (69.9)	91.70	39.0 (2.69)	1.54
10	3.00 (76.2)	106.5	26.5 (1.83)	1.07
11	4.00 (102)	114.3	31.5 (2.17)	0.797

† 1 bar = 10^5 N mm^{-2} = 14.50 psi.

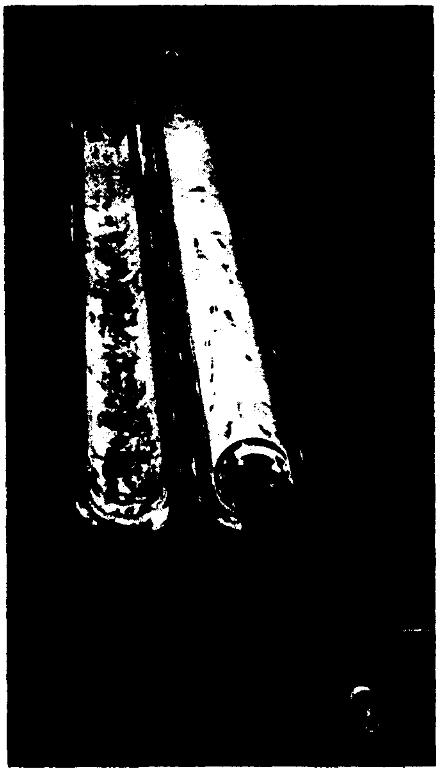
Table 2. Stainless steel specimen properties and confined propagation pressures measured

	D in. (mm)	$\frac{D}{t}$	P_{pc} psi (bar†)	$\frac{P_{pc}}{\sigma_0} \times 10^3$
1	1.002 (25.45)	22.99	2103 (145.0)	44.37
2	1.002 (25.45)	29.04	1258 (86.76)	27.35
3	1.253 (31.83)	34.81	778 (53.66)	23.22
4	1.001 (25.43)	35.75	808 (55.72)	17.96
5	1.002 (25.45)	48.87	422 (29.10)	11.25
6	1.504 (38.20)	49.97	524 (36.14)	11.93
7	1.502 (38.15)	51.79	321 (22.14)	7.52
8	1.374 (34.90)	68.70	317 (21.80)	2.73
9	1.495 (37.97)	71.19	126 (8.690)	2.72
10	1.750 (44.45)	77.78	128 (8.828)	2.84
11	1.500 (38.10)	93.75	77 (5.310)	1.56

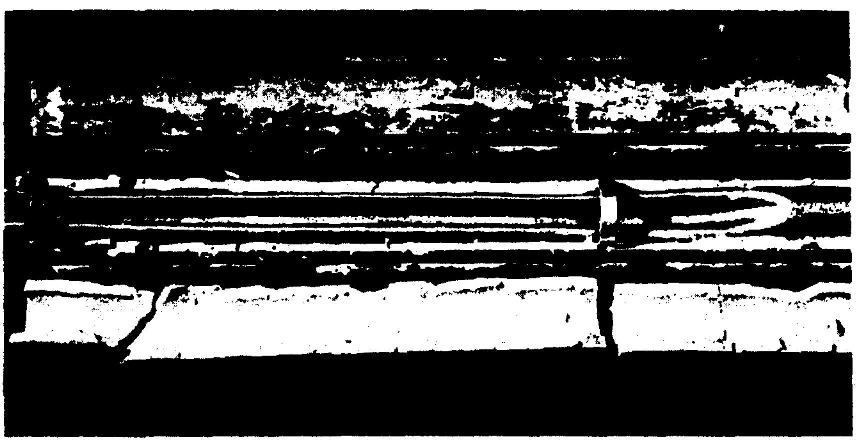
† 1 bar = 10^5 N mm^{-2} = 14.50 psi.

Table 3. Additional material properties of test specimen (mean values)

	Young's modu. (E) psi (N mm^{-2})	Hardening para. (n)
Aluminum 6061-T6	9.8×10^6 (67.6×10^3)	31.5
Stainless steel 304	26.8×10^6 (185×10^3)	13.6
Mylar	0.735×10^6 (5.07×10^3)	—



(a)



(b)

Fig. 4. Test specimen after experiment.

Fig. 5. Profile of confined propagating buckle (D/t = 25.8, AL-6061-T6).

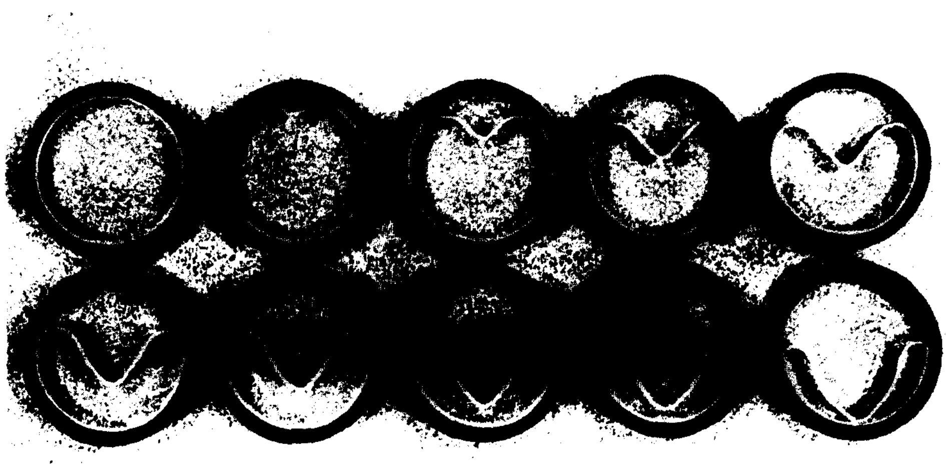


Fig. 6. Sections through profile of confined propagating buckle (D/t = 53.4, AL-6061-T6).

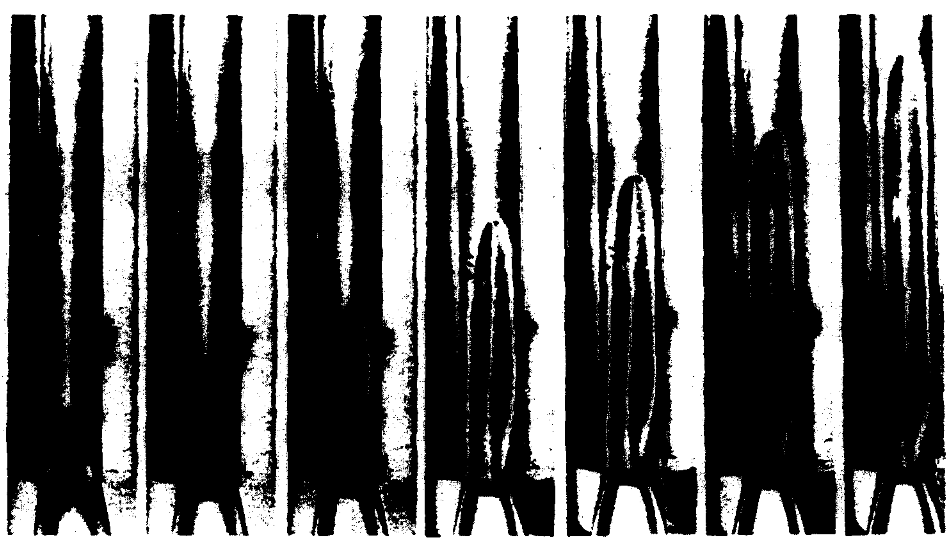


Fig. 7. Confined buckle propagation (D/t = 53.4, AL-6061-T6).

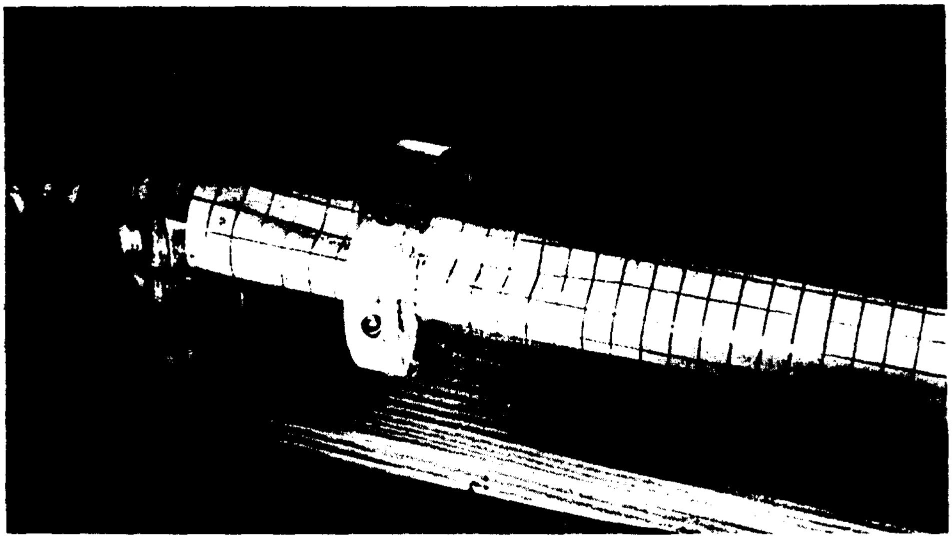


Fig. 9(a). Initiation process.

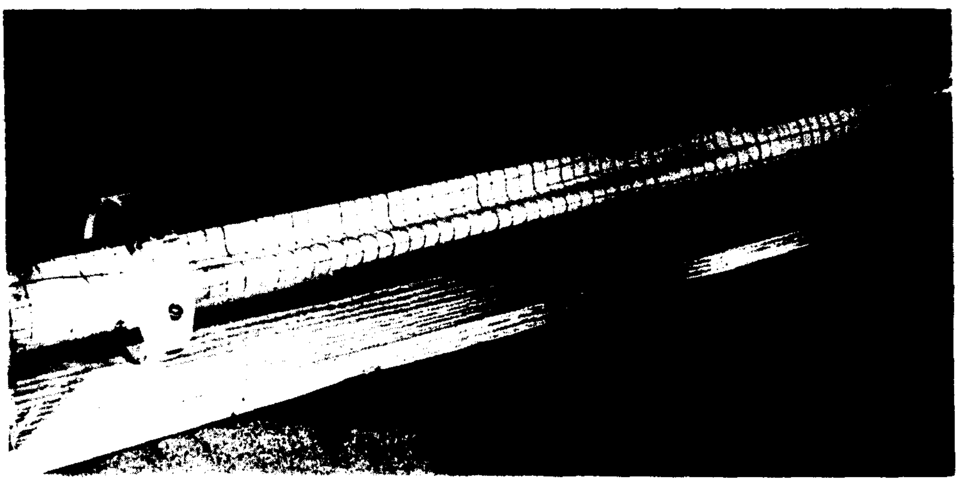


Fig. 9(b). Steady-state propagation of a buckle in a confined thin-walled mylar shell ($D/t = 182$).

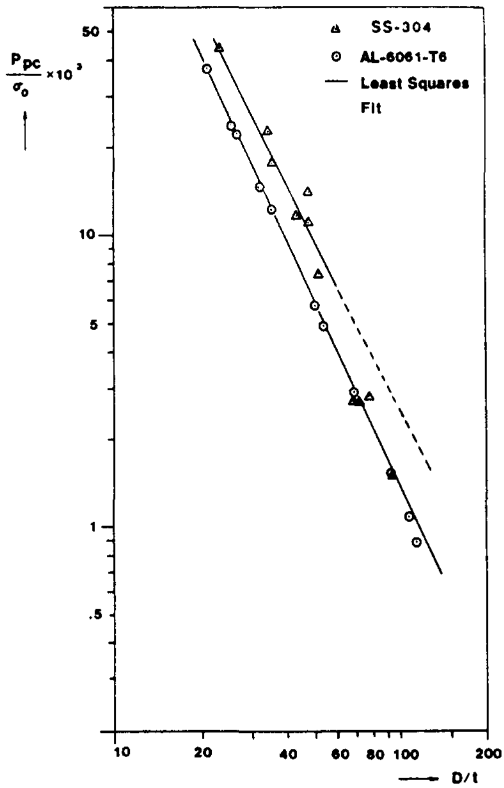


Fig. 8. Experimental confined propagation pressures as a function of D/t.

power law. The values of A and β obtained are as follows:

Material	A	β
Aluminum†	26.07	2.15
Stainless steel‡	18.29	1.92

† Specimens 1–9, Table 1.
‡ Specimens 1–7, Table 2.

In order to demonstrate that the phenomenon is strongly influenced by geometric factors, separate experiments were performed in which thin-walled mylar tubes were tested. For this material, the phenomenon takes place strictly in the linearly elastic regime. The experimental set-up was very similar to the one used for the metal tubes. Due to the flexible nature of these tubes, a solid mandrel was used to support the structure during the molding stage. The mandrel was later removed and the tube sealed at both ends. The tubes had a mean diameter of 1.82 in. (46.3 mm) and wall thickness of 0.010 in. (0.25 mm). The measured confined propagation pressure was 0.55 psi (0.038 bar). As a result, the experiment was carried out by evacuating air in a controlled fashion from inside the tube.

The mechanism of buckle propagation was very similar to what has been described for metals. This is in sharp contrast to the behavior of propagating buckles described in [14–19] which only occur if the material stress–strain curve is nonlinear. The reasons for this difference will be explained below.

A demonstration of the initiation and propagation process of such a buckle in a thin mylar elastic shell confined in a transparent acrylic tube is shown in Fig. 9. The characteristic U shape of the buckled tube is clearly visible. The length of the transition regime can be seen to be substantially longer than that of Fig. 5.

As stated earlier, if a buckle is initiated in a constant pressure environment at a value of pressure higher than the confined propagation pressure of the tube, the buckle will accelerate to steady state in a few diameters and will proceed to propagate at a constant velocity. These comments come from observations made in the lab. in the course of the experimental program described above. No measurements have as yet been carried out under dynamic conditions.

MODELING OF THE PROBLEM

Prediction of the steady state quasistatic propagation of a confined buckle by analytical means is rather complicated. A complete formulation of the problem must deal with the large deflections exhibited during the collapse process (see Figs 5 and 6), the inelastic nature of the tube material, the stability issues involved and the inherent contact problems. In the case of the familiar problem of a propagating buckle in offshore pipelines[18, 19], a great deal of insight to the problem was developed by examining the large deflection collapse of a ring (or long tube section) under external pressure (see [22, 23]). Encouraged by this success, a similar approach is followed for the confined propagating buckle.

The problem solved[13] consists of a thin, inextensional ring confined in a rigid, smooth, contacting, circular cavity (see Fig. 10). The ring has a small initial geometric imperfection which causes it to be locally detached from the confinement. The cavity formed is pressurized with fluid pressure (P). A numerical solution procedure was developed for studying the progressive collapse of the ring under this pressure. The problem was formulated through a large deflection nonlinear analysis. Three different material models were examined: linearly elastic, nonlinear elastic and elastic–plastic models.

Placed in this context, the problem reduces to a nonlinear two-point boundary value problem which was solved numerically, using an incremental procedure. Details of the solution can be found in [13]. A sequence of collapse configurations for such a ring are shown in Fig. 11. The associated pressure-change in volume response is shown schematically in Fig. 12. In the presence of a small initial imperfection, the response is characterized by a sharp rise to a limit load and a sharp drop down to a pressure plateau at which most of the deformation occurs. After the first point on the ring circumference touches the opposite wall, the response becomes stable again as shown in the figure.

For the linearly elastic case, the limit load is strictly a function of the initial imperfection. It is affected by both the amplitude and the shape of imperfection used. For the idealized inextensional ring case studied, the perfect geometry case is predicted to have no buckling load. The stable (zero volume change axis) and unstable branches only cross at an infinite value of pressure (see [7]). This idiosyncrasy of the problem as formulated will be avoided

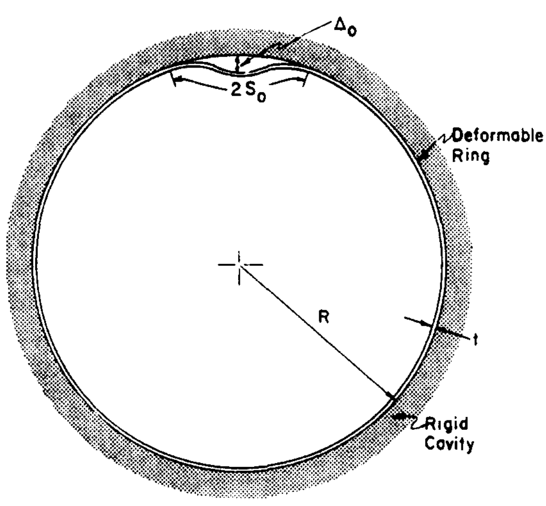


Fig. 10. Initial geometry of inextensional confined ring.

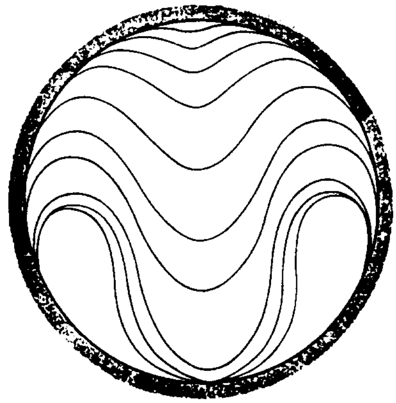


Fig. 11. Confined ring collapse configuration sequence (linearly elastic case).

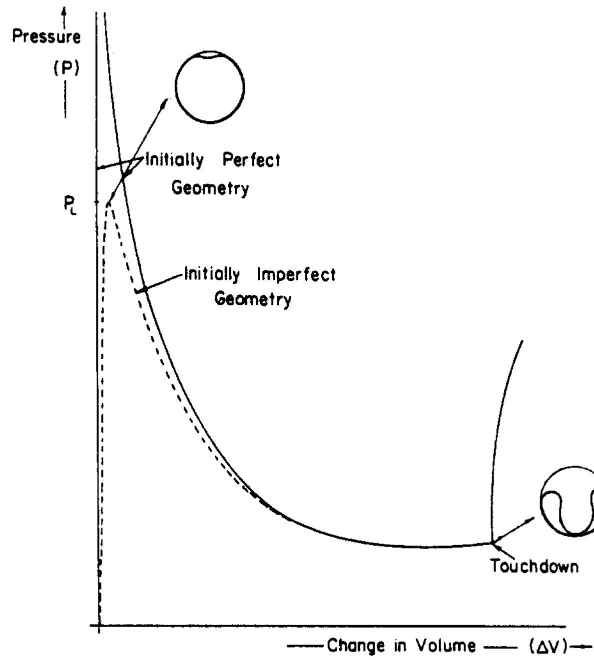


Fig. 12. Prebuckling and postbuckling response of confined ring under external pressure.

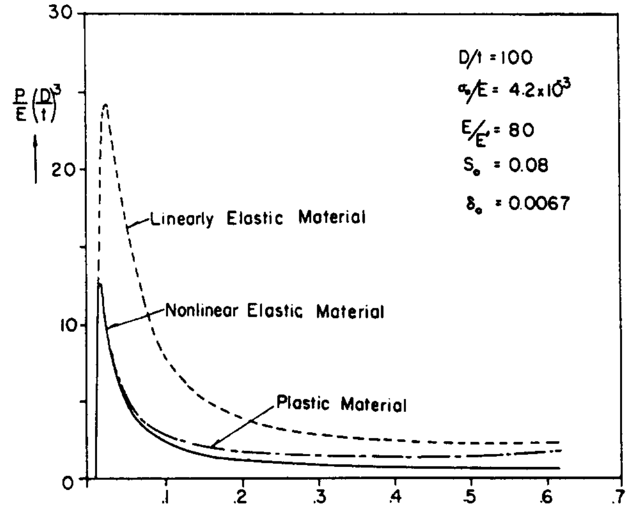


Fig. 13. Pressure vs crown displacement for different material models.

in what follows by considering rings to always have a small initial imperfection of fixed geometry.

Figure 13 shows how the crown displacement varies with pressure. Results for linearly elastic, nonlinear elastic and elastic–plastic materials are included. For the last two, a linearly strain hardening material with strain hardening modulus (E'), 80 times less than the Young's modulus, was used. The nonlinearity in the stress–strain behavior causes a reduction of the limit load. In addition, differences in the unstable response are observed. However, the general characteristics of the response remain common to all material models. It is also interesting to observe the similarity in the predicted collapse configurations in Fig. 11 and those prescribed by the profile of the confined propagating buckle in Fig. 6.

From the above discussion, it is clear that the maximum pressure an initially imperfect confined ring (or long tube) can sustain is represented by the limit pressure, P_L (Fig. 14). For pressures lower than P_L, the ring is stable. However, it is observed that for each value of pressure $P_m < P < P_L$ three equilibrium states are possible. Under constant pressure conditions, the ones on the extreme left (A) and right (C) of the response are stable. The intermediate one (B) is unstable. Thus, if under constant pressure conditions equilibrium state A is adequately disturbed, the ring will snap to equilibrium state C. This characteristic of the response is fundamental to propagating buckle problems (see [17–19]).

In the case of a long confined tube, pressurized externally, it is intuitively obvious that the collapse process will be initiated at the section with the biggest local imperfection. Once the local limit pressure is reached, the section locally buckles. Further collapse of the tube will depend strictly on whether the applied pressure is subsequently higher or lower than the confined propagation pressure of the tube. If the pressure is at or above P_{pc}, then the locally buckled section acts as the disturbance that progressively forces the tube section adjacent to it to jump from equilibrium state A to equilibrium state C in the process collapsing that section. The profile of propagation can in fact be viewed as a series of rings of varying degrees of deformation (see Fig. 6), each one assisting the one downstream from it to cross the energy barrier between A and B.

For certain classes of materials, the pressure-change in volume response can be used in conjunction with the energy argument developed in [18] to establish the confined propagation pressure of the tube. For the purposes of this analysis, it is convenient to distinguish between elastic path-independent material behavior and inelastic path-dependent behavior.

(a) *Elastic material*

Consider the steady state propagation of a confined elastic tube. At a critical pressure,

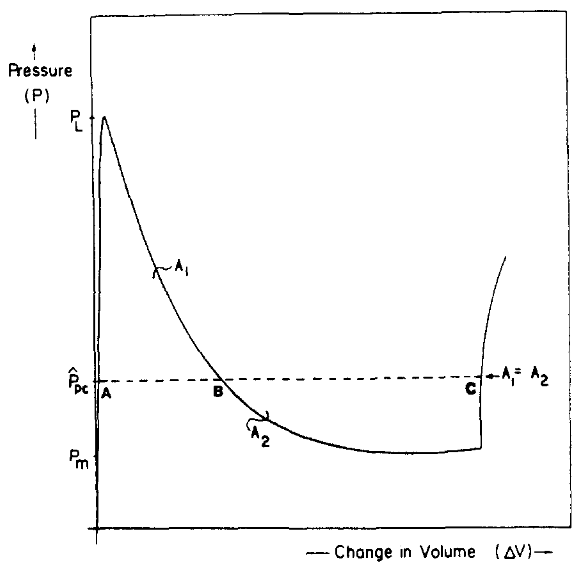


Fig. 14. The Maxwell construction for path independent materials.

P_{pc}, the buckle propagates quasistatistically. Let the buckle propagate by a unit length. Thus, a unit length of the tube crosses from A to C in Fig. 14. In doing so, the collapsed section reduces its volume by

$$(\Delta V_C - \Delta V_A).$$

Thus, the work done by the external pressure in the process is given by

$$\hat{P}_{pc}(\Delta V_C - \Delta V_A).$$

For an elastic material, the work done during a deformation process is independent of the loading path and depends strictly on the initial and final states. State C, in Fig. 14, can be reached from A by moving along the path calculated (i.e. up to the limit load, down to P_m and up to C); or by moving along path ABC (constant pressure path). The latter represents the propagation process. Since the first has already been calculated, it can be used to calculate the energy absorbed by the unit length of collapsed tube. Thus, equating internal and external work done, the following expression can be written for $\hat{P}_{pc}$:

$$\hat{P}_{pc}(\Delta V_C - \Delta V_A) = \int_{\Delta V_A}^{\Delta V_C} P(\Delta V)\,\mathrm{d}\Delta V. \tag{2}$$

It is easy to show that expression (2) requires that $\hat{P}_{pc}$ be at a level which makes areas A_1 and A_2, in Fig. 14, equal (Maxwell line[18]).

The pressure-change in volume response of a confined linearly elastic ring having modulus E, diameter D and wall thickness t is shown in Fig. 15. This solution suitably extended to a plane strain tube, by scaling the pressure by the factor $(1-\nu^2)^{-1}$ (ν being the Poisson's ratio of the material), was used in the procedure described to calculate a value for the confined propagation pressure $\hat{P}_{pc}$. It was found that

$$\hat{P}_{pc} = \frac{3.64E}{(1-\nu^2)}\left(\frac{t}{D}\right)^3. \tag{3}$$

Equation (3) predicts $\hat{P}_{pc} = 0.5$ psi for the mylar tube tested (Table 3). This is 10%

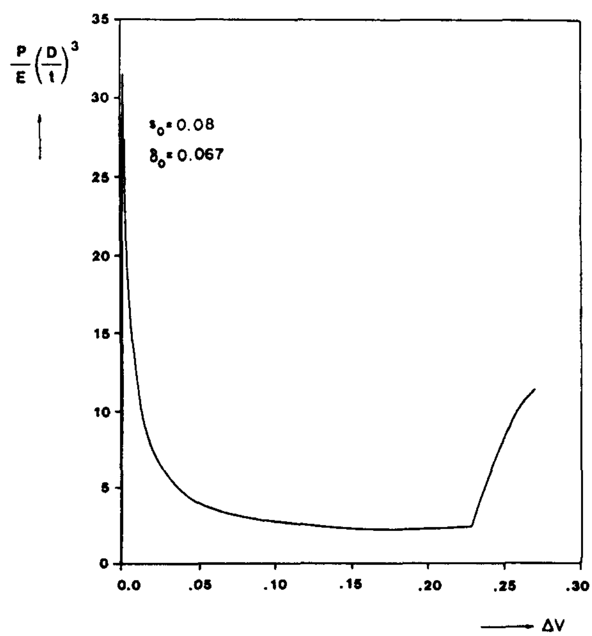


Fig. 15. Pressure vs volume change (linearly elastic case).

lower than the value measured. It must be emphasized that the measured propagation pressures of these tubes are more susceptible to errors introduced by the presence of friction and other factors due to the very low value of the propagation pressure.

The same procedure can be used for calculating the confined propagation pressure of nonlinear elastic tubes.

(b) *Elastic–plastic material*

All metal tubes tested exhibited plastic irreversible deformations. Plastic material behavior is loading path dependent. Loading path independence is a necessary condition for the application of the energy balance argument developed above. However, for certain loading paths (proportional or "nearly" proportional stress histories), the assumption of path independence of material behavior is correct even for elastic–plastic materials. References [18, 19] showed that, in the case of a propagating buckle in a long tube under external pressure, the profile of propagation induces nearly proportional loading paths; as a result, the energy balance argument mentioned was used to obtain a good engineering approximation to the propagation pressure.

Unfortunately, that success was not repeated in the case of the confined propagating buckle. An indication that this problem may be more complicated is given by the large difference, exhibited in Fig. 13, between the response from a bilinear elastic material and a multilinear inelastic material. This difference is due to the fact that large sections of the circumference undergo unloading and reverse bending after being bent into the plastic range. Examination of the profile of confined buckle propagation (Figs 5 and 7) clearly indicates that the buckled section also undergoes rather severe bending and stretching in the axial direction. The membrane and bending deformations lead to the conclusion that the loading paths, in a good part of the shell, are non-proportional. As a result, the energy balance argument fails.

DISCUSSION OF RESULTS

The experimental program described, demonstrated that under constant pressure conditions a long cylindrical confined shell can develop a propagating buckle. The buckle propagates within the confining cylindrical cavity and collapses the shell to a degree that renders it useless. A critical pressure has been identified, below which the buckle will not propagate. This pressure has been given the name confined propagation pressure. A limited parametric study indicates a power law relationship between P_{pc} and the tube D/t as follows:

$$P_{pc} \propto \left(\frac{D}{t}\right)^{\beta}. \tag{4}$$

For material and geometric combinations for which the phenomenon occurs in the elastic range, the value of β in (4) is -3. For combinations governed by inelastic effects, a smaller, well-identified value of β has been obtained (-2.15 for AL-6061-T6, -1.92 for SS-304). For each material, an intermediate range of D/t value exists for which β smoothly changes values from -3 to the ones quoted above. It seems that test specimens 8–11 in Table 2 belong to this intermediate range of D/t values for stainless steel 304; this would explain the observed deviation of these results from the power law developed (see Fig. 8).

The phenomenon has been explained by observing the pressure-deflection response of a confined ring. For elastic materials, it has been shown that this response can form the basis for the exact prediction of P_{pc}. The energy balance argument on which this solution was based was found to be inappropriate for elastoplastic materials.

Finally, we return to Fig. 1. In view of the findings of this work and the length of the damaged casing reported in [21], it is suggested that the casing in question was buckled by a confined propagating buckle. It is important to note that for typical steel shells having D/t values less than 50, the confined propagation pressure can be substantially lower than the collapse pressure of the unconfined tubes which is usually the design pressure for oil well casing.

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